A Cumulant-based stock market volatility modeling – Evidence from the international stock markets

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ABSTRACT

The pourpose of this paper is to propose the Stock Market (SM) volatility estimation method based on the Higher Order Cumulant (HOC) function, and to apply it to the cases when stock market returns have a non Gaussian distribution and/or when a distribution of SM innovations is unknown. The HOC functions of the third and fourth order are used not only as a means for non Gaussian model testing but also as sufficient statistics, which is indispensable in estimating the AR and MA parameters of the squared SM returns. The empirical analysis is based on the daily closing values of the SMI, DJIA, SP500, DAX, FTSE100, NASDAQ and BSE indexes. The time horizon includes the period between March 30, 2010 and February 6, 2013. ARMA parameter estimation is performed by using the well known GARCH algorithm from Eviews, as well as the estimation algorithm based on higher order cumulant (HOC) functions, which is introduced in this paper. Ultimately, the Hinich portmanteau statistics are used to test the adequacy of ARMA-GARCH and ARMA –HOC models. The research outcome demonstrates that ARMA-HOC model produces independent innovations and captures the model dynamics while the ARMA -GARCH model fails to do it. All data are taken from Bloomberg.

Keywords: Volatility modeling, GARCH model, ARMA-HOC model, Non Gaussian innovations, Higher Order Cumulant function, Swiss Market Index.

1. INTRODUCTION

Ever since the GARCH (Generalized Autoregressive Conditional Heteroschedastic) paradigm started dominating the field in the area of Stock Market (SM) volatility forecasting, a cohesive body of GARCH literature has encapsulated many of the aspects of its ability to capture the Stock Market (SM) stylized facts. Unfortunately, the GARCH paradigm, led to deeply embedded illusions of its ability to produce consistent and efficient estimates of the stock market volatility. The unfavorable issues of the GARCH volatility prediction are progressively increasing at each step of its application: the erroneous assumption that standardized residuals are independent and identically distributed; the failure of the GARCH model variants to capture the stylized facts coupled to stock market returns when their innovations do not give evidence for assumption of any of the known probability distributions; the pre-set GARCH idea to use prediction of squared returns as a proxy for the volatility forecast and to estimate it by using only the second order moments – autocorrelation function of SM squared returns and ultimately kurtosis.

Today the state of art of SM volatility forecasting improvement offers two statistically different paths, which lead either to a new analytical forms of the GRACH model or to the discovery of a new model testing method based on kurtosis and autocorrelation function.
The first research direction has been widely explored in the last three decades by hold-outs of the GARCH paradigm. Insofar, the popular ways of comparing volatility models have been: to compare Mean Forecast Errors (MFE) or to use AIC or BIC criteria. While numerous studies have compared the forecasting abilities of the historical variance and GARCH models, no clear winner has emerged. In a scrupulous review of 93 such studies, Poon and Granger (2003) reported that 22 find that historical volatility forecasts future volatility better out-of-sample, while 17 studies find that GARCH models forecast better. Brooks, Burke and Persand (2001) used DJ composite daily data to test in- and out-of-sample forecasts obtained with GARCH, EGARCH, GRJ and HS (historical volatility) models. The coefficient of determination (R^2) achieved was around 25% for each of the models.

Ideally, a good volatility model should have the capacity to capture decreasing autocorrelations of squared observations. Carnero, Peña & Ruiz (2004) compared the Autoregressive Stochastic Volatility (ARSV) model and the GARCH model using the kurtosis – autocorrelation relationship in squared returns as their benchmark. No conclusive results were obtained.

Malmsten &Teräsvirta (2004) discussed the stylized facts of SM returns in connection with the GRACH and ARSV model. Their paper contains an application of a novel method of obtaining confidence regions for the kurtosis-autocorrelation relationship. The exact representation of kurtosis is derived for both GARCH and stochastic volatility models. It was demonstrated both analytically and empirically that the GARCH (1, 1) model with high starting autocorrelation of squared returns was observed in a large number of financial series. Thavaneswaren at all (2009) derived the kurtosis of the AR model with random coefficients and GARCH intonations. It was also found that “the first-order autocorrelations”, given the certain kurtosis, are lower in the ARSV than the EGARCH model with normal errors. This may, at least to a certain extent, explain the fact the ARSV (1) model seems to fit the data better than its EGARCH or GARCH counterpart. However, the skewness of the squared returns which is frequently encountered in stock market variables cannot be reproduced by any of the existing volatility models.

The conclusion that emerges from those considerations, which are largely based on results about the kurtosis structure of these models, is that "none of the models dominates the others when it comes to reproducing stylized facts in typical financial time series". By comparing the difference between the theoretical GARCH kurtosis and the estimated kurtosis, it is demonstrated how t- distribution assumption adds to the flexibility of the GARCH model and helps the model parameters to reproduce, in a better way, but not completely, the stylized fact of high kurtosis and relative low autocorrelations of squared observations. Partial empirical improvements are made only when considering t or GED distribution of squared returns.

Starica (2003) investigated how close the simple endogenous dynamics imposed by the GARCH (1, 1) process is to the true dynamics of returns of the stock market indexes, the Standard & Poors 500 (S&P 500) and the Dow Jones Industrial Average (DJIA) index. The results lead to the rejection the hypothesis that a GARCH (1, 1) process is the true data generating process of the longer sample of returns of the S&P 500 stock market index.

Both volatility clustering and conditional non-normality can induce the leptokurtosis which is typically observed in financial data. Bai, Russell & Tiao (2003) found theoretically and empirically that, for GARCH and AR Stochastic Volatility models, volatility clustering and non-normality contribute interactively and symmetrically to the overall kurtosis of the series of squared returns.

The second path of the SM volatility research was initiated by Hinich (1996). He introduced the rigorous model testing criteria: H test. The test is in fact the extension of the Box-Pierce test (1970 a) and is based on third order sample cumulants, called bicorrelations by Hinch. Additionally, Lim, Hinich and Liew (2006) employed the Hinich portmanteau bicorrelation test as a diagnostic tool to determine the adequacy of the GARCH models for eight Asian stock markets. They showed that that the null hypothesis of the independent innovations is strongly rejected. Their findings question the applicability of the OLS estimation method, which is consistent if the error time series {et} is a martingale difference, which happens only if the third order triple correlations are zero for all lags except for the lag (0,0).

This article is concerned only with the second pathway of the volatility research. It states that, insofar the sufficient statistics for ARMA- GARCH parameter estimation is considered to be the autocorrelation function with addition of the kurtosis solely; so far the existing parameter estimation methods will not be applicable to non Gaussian returns and will not produce independent innovations. Accordingly it proposes a paradigm change focused on ARMA model parameter estimation instead only on model testing as has been done so far.

In general, the introduction of a new paradigm is very difficult. Since paradigms are so hard to change, there is a natural tendency to dismiss all evidence that does not fit into the existing framework. However the evidence and experiences with the inadequacies of GARCH model, which have multiplied themselves in the last decade and the advances in using higher order cumulants in digital signal processing (Al-Smadi and Wilkes, 2002), wireless communications, speech processing and in EEC modeling (Goshvarpour at all 2012), made it possible to develop a cumulant based approach to the non Gaussian volatility model building.

The aim of the paper is twofold: to trigger the paradigm shift by stressing the importance of applying the third and the forth order cumulant functions for the ARMA-HOC parameter estimation and to show that ARMA- HOC estimation method captures the SM stylized facts better while producing innovations which are independent. Both HOC-ARMA and GARCH – ARMA parameter estimations are obtained by using a daily closing prices of the SMI, DJIA, SP500, DAX, FTSE100, NASDAQ and BSE indexes, for the period between March 30, 2010 and Feb.6, 2013, taken from Blumberg. Ultimately, the Hinich test , which are based on the third and the fourth order cumulants of the ARMA innovations, are used to show that the proposed HOC based estimation method does capture the SMI index stylized facts and produces independent forecasting errors.

The paper is organized as follows: The ARMA –GARCH and the ARMA HOC models are theoretically described in Section 2. The higher order cumulant functions are defined in the same section and used to introduce an extension of the Yule –Walker method which can be applied in the case of non-Gaussian AR parameter estimation. The same section presents a cumulant based MA-HOC parameter estimation method. Further on, in Section 3 it is tested whether the hypothesis that states that the HOC-ARMA parameter estimation is more successful in extracting the information about the stock market stylized facts, contained in the squared returns. Since the stylized facts are not seen only in autocorrelation function, kurtosis and skewness of squared returns, but also in the third and the fourth order cumulant functions mentioned above, the fourth section presents the empirical ARMA-GARCH and ARMA HOC models and their residual analysis. It is shown that the parameter estimation, based on HOC functions, flatten, more successfully, the second, the third and the fourth order cumulant functions of the squared returns. The final section presents the conclusion and suggestions for further research.

2. THE PROBLEM AND THE MODEL

Statistical properties of stock market returns, which are common across a wide range of developed stock markets and time periods, are called stylized facts. Stylized statistical properties of asset returns of developed markets are analyzed empirically and subsequently summarized by Cont (2001).

The fact that market returns are often characterized by volatility clustering, which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility, implies that the past volatility could be used as a predictor of the volatility in the next periods. As an indication of volatility clustering, squared returns often have significant autocorrelations and consequently can be modeled by using the well known GARCH model.

2.1. The GARCH-ARMA model building

Let e_t denote a discrete time stationary stochastic process. The GARCH (p, q) process is given by the following set of equations

$$r_t = \log(p_t/p_{t-1}) \tag{1}$$
$$r_t = x(_k)g(_k) + e_t \tag{2}$$
$$e_t = v_t\sqrt{h_t}$$
$$e_t/_{t-1} \approx N(0, h_t) \tag{3}$$
$$h_t = \alpha_0 + \sum_1^p \alpha_i e^2_{t-i} + \sum_1^q \beta_j h_{t-j} \tag{4}$$

where $\{p_t\}$ represents stock prices; $\{r_t\}$ represents random returns; $\{e_t\}$ represents de-trended returns; $x(_k)$ is a vector of explanatory variables; $g(_k)$ is a vector of multiple regression parameters; h_t is the conditional volatility; α_i is autoregressive; and β_j is the moving average parameter as related to the squared stock market index residuals. An equivalent ARMA representation of the GARCH (p, q) model (Bollerslev, 1982, pp. 42-56): is given by:

$$e_t^2 = \alpha_0 + \sum_1^p(\alpha_i + \beta_i)e^2_{t-i} + \nu_t - \sum_1^q \beta_j\nu_{t-j} \qquad (5)$$

where $\nu_t = e_t^2 - h_t$ and, by definition, it has the characteristics of (i.i.d) white noise. h_t is known as GARCH variance.

In this context, the GARCH (p, q) volatility model is simply an Autoregressive Moving Average -ARMA (p, q) model in e_t^2 driven by i.i.d noise ν_t, which is Gaussian random variable. It is worth stressing that the GARCH variance, h_t, in time series analysis, appears to be merely an estimate of the squared de-trended SM returns e_t^2.

The best known Gaussian ARMA model building methodology is known to be the Box-Jenkins (B-J) iterative methodology, which includes three steps: model order determination, parameter estimation and model testing (Box and Jenkins, 1970).

The B-J methodology assumes that each stationary time series can be treated as an output from the AR(p), MA(q) or ARMA (p,q) filter, which has as an input uncorrelated and Gaussian innovations, known as "white noise" $\{\nu_\tau\}$.

The ARMA model has the following form: $A(Z)\, e_t^2 = B(Z)^* \,\nu_t$, where Z is a backward shift operator:

$e_{t-1}^2 = Z^{-1} e_t^2 : e_{t-k}^2 = Z^{-k} e_t^2$, and where $A(Z) = 1-\alpha_1 Z^{-1} - \alpha_2 Z^{-2} - \alpha_p Z^{-p}$ and $B(Z) = 1-\beta_1 Z^{-1} - \beta_2 Z^{-2} - \beta_q Z^{-q}$, are characteristic transfer functions of orders p and q respectively. The roots of the characteristic functions of the ARMA model must be within the unit cycle to guarantee stationarity and invertibility of the model.

Model Order determination can be accomplished by using several criteria : Final prediction Error, (FPE) Akaike information Criterion (AIC) or Minimum Description Length (MDL) based on covariance matrix (Liang at all , 1993).

Given the fact that driving noise is assumed to be i.i. distributed, parameter estimation method to be used is either maximum likelihood or minimum variance or ordinary least squares method. Sufficient statistics for ARMA parameter estimation in this case includes the first and the second order moments e.g. mean variance and autocorrelation function.

The model testing is accomplished by using Q statistics based on the autocorrelation function of innovations, $C_v(l)$, $l=1,2\ldots L$, as suggested by Box&Pierce (1980). It shows how successfully the model performs a “digital whitening” of observed time series and has the form:

$Q \approx n \sum_1^L C^2_v(l)$ which has $\chi 2$ distribution with d.f. is L-p -q, where L is max lag (6)

As proved empirically, in the case of stock market returns, driving noise is not independent and most usually, it is non Gaussian either. Subsequently the second order moment and correlation function do not represent a “sufficient statistics”, either for the ARMA parameter estimation, or for the model testing. In fact, it is well known that for a non-Gaussian process, the higher order moments exist and are different from zero. However, this result failed to imminently inspire a concerted effort between the hold-outs of the old paradigm to develop a better theory for the SM volatility modeling.

Nonetheless, it is worth mentioning the case of using Gram Charlie probability density function (pdf) of innovations, which is considered in reevaluating the GRACH ability to represent the SM data generating process (Christodoulakis G. and Sathell S. 2007). It is theoretically proved that the pdf of the SM squared returns does not depend on skewness but does depend on kurtosis. It is argued that for unbiased estimators, non-normality increases the discrepancy between the true and the estimated forecast error statistics. In other words, the article suggests solution for the GARCH evaluation problem while keeping the GARCH paradigm as it was originally introduced. Instead, this article shifts the emphasis from a model testing to the problem of ARMA parameter estimation of non Gaussian SM squared returns.

2.2. The HOC-ARMA Parameter Estimation

Danish statistician Thiele (1898) realized that the normal distribution was unsatisfactory for describing economic and demographic data and proposed the use of a special type of distribution to represent a new system of skewed distributions f(z) by using the quantities of the distribution expressed as $\lambda_r = \kappa_{r+2}/\kappa_2^{(r+2)/2}$. Thiele interpreted cumulants κ_r as follows: "The mean $\kappa 1$ depends on location and scale, the variance k_2 depends on scale but is independent of the location, while λ_r are independent of both location and scale and therefore describe the shape of the distribution" (Hald 1981, p. 7). Since then, at least three forms of a general probability density distribution with a priory unknown shapes were proposed: Chebishev, Gram Charlie and Edgeworth. Insofar the best properties in terms of integralability and convergence are found in Edgeworth distribution approximation. Its form allows any standard probability distribution f (z) to be expressed in terms of Gaussian distribution ϕ (z):

$$f(z) = \phi(z) + -\lambda_3\phi^3(\zeta)/3! + \lambda_4\phi^4(z)/4! \ + \lambda_3{}^2\phi^6(z)/6!.... \quad (7)$$

where $z = (x-\kappa 1)/\sqrt{\kappa 2}$.

Recently, Zhang, Mykland & Aït-Sahalia (2011) proved that the empirical distribution of realized volatility (RV) can be described by using Edgeworth expansion and developed a general method for the computation of the cumulants.

Given the fact that the second order cumulant function (correlation function) is phase blind, higher-order cumulants have gradually become a powerful tool for a phase estimation in digital signal processing, harmonic retrieval, non Gaussian signal reconstruction , detecting and characterizing the properties of nonlinear systems ,biomedicine and image reconstruction theory and hence widely used in many diverse fields. An excellent review of the HOC results is provided by Mendel (1991). Incredibly as it may seem, the HOC function application in stock market volatility parameter estimation and GARCH order determination have not be explored insofar.

An extension of the well known Yule-Walker equations used for Gaussian time series models, in terms of higher order cumulants for non Gaussian autoregressive models is theoretically developed by JammalamadakaS.R., T. Subba R. &Tterdik Y (1991). In the area of digital signal processing, Giannakis (1990) was the first to show that AR parameters of non-Gaussian ARMA signals can be estimated cy calculating the third- and the fourth-order cumulant function of the output time series, as following:

$$C^3{}_x(r,s = (\sum(x(t)x(t+r)x(t+s))/n, \qquad r=1,2..L \ ,s=1,2...L \quad (8)$$

$$C^4(r,s,v,) = (\sum(x(t)x(t+r)x(t+s)\ x(t+v))/n \quad (9)$$

where n is a number of observations and where the second-order cumulant $C^2_x(\tau)$ is just the autocorrelation function of the time series x_t.

The AR parameter estimation for non-Gaussian ARA (p,q) processes is based on the modified Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants (Giannakis ,1990):

$$\sum_1^p \alpha_i C^3(k-i,k-l) = - C^3(k,k-l) \qquad k \geq l \geq q+1 \tag{10}$$

$$\sum_1^p \alpha_i C^4(k-i,k-l, k-m) = - C^4(k,k-l, k-m) \qquad k \geq l \geq m \geq q+1 \tag{11}$$

Or, in the matrix form:

$$\begin{vmatrix} C^3(q+1-p,k) & C^3(q+2-p,k) & \ldots C^3(q,k) \\ C^3(q+2-p,k) & C^3(q+3-p,k) & \ldots C^3(q+1,k) \\ C^3(q,k) & C^3(q+1,k) & \ldots C^3(q+p-,k) \end{vmatrix} \begin{vmatrix} \alpha(p) \\ \alpha(p-1) \\ \alpha(1) \end{vmatrix} = \begin{vmatrix} -C^3(q+1,k) \\ -C^3(q+2,k) \\ -C^3(q+p,k) \end{vmatrix}$$

$$\mathbf{C}^3_k * \mathbf{a} = - \mathbf{c}^3_k \tag{12}$$

Gannakis (1990) also showed that the matrix $\mathbf{C}^3_k$ is of full rank and thus provide a unique solution for the parameters αi, i=1, 2...p.

The efficient MA parameter estimation can be performed by applying one of the proposed algorithms, for instance, q-slice algorithm made by Swami A. & Mendel J. (1989). Q-slice algorithm uses autoregressive residuals $\{ar_t\}$, calculated after estimating the AR parameters of the ARMA model (12). Following up, the impulse response parameters ψ_i of the pure MA model of ar_t residuals are then estimated by using cumulants (8 and 9):

$$ar_t = \sum_1^\infty \psi_j \, a_{t-j} \qquad i=1.2\ldots\infty \tag{13}$$

$$\psi j = \frac{\sum_1^p \alpha_i C^3(q-i,j)}{\sum_1^p \alpha C^3(q-i,0)} \qquad j=1,2\ldots q \tag{14}$$

Or by using :

$$\psi j = \frac{\sum_1^p \alpha_i C^4(q-i,j,0)}{\sum_1^p \alpha_i C^4(q-i,0,0)} \qquad j=1,2\ldots q \tag{15}$$

The MA parameters of the ARMA model are obtained by means of the well known relationship (Box G.E.P. 1970):

$\beta_j = \sum_0^p \alpha i \psi_{(j-i)}$ j=1,2…q (16)

The cumulants based ARMA estimates are shown to be asymptotically optimal (Friendler B. & Porat B, 1989).

2.3 HOC- ARMA Model Testing

The test statistics suggested by Hinich (1996) is aimed to test serial dependence in the data by using auto correlation, bicorrelations and tricorrelations. The null hypothesis is that the innovations are realizations of a stationary pure white noise process. Therefore, under the null hypothesis, all $C^2(r)] = 0$, for all $r \neq 0$, the bicorrelations $C^3(r, s) = E[\nu(t)\nu(t-r)\nu(t-s)]$ for all r and s , except where r =s=0 , and the tricorrelations $C^4(r, s, v) = E[\nu(t)\nu(t-r)\nu(t-s)\nu(t-v)] = 0$ for all r ,s and v , except where r = s = v = 0

The H2 statistics, known as Q statistics, originally developed by Box-Pierce (eq. 6), is used to test linear serial dependence. H3 and H4 are designed to test for the existence of a higher order serial dependence (Wild, Foster and Hinich, 2010, pg 9):

$H3=(n-s)\sum_{s=2}^{L}\sum_{r=1}^{s-1}\{C^3(r,s)\}^2 \approx \chi 2$ with L(L-1)/2 d.f. , where L is number of lags. (17)

$H4=(n-v)^{3/2}\sum_{v=2}^{L}\sum_{s=2}^{v-1}\{\sum_{r=1}^{s-1}\{C^3(r,s,v)\}^3 \approx \chi 2$ with L(L-1)(L-2)/3 d.f. (18)

The number of lags L is defied as $L=n^b$, with 0<b<.5 , for the H2 and H3and <0<b<.33 for the test based on the forth order cumulants.According to Wild , Foster and Hinich (2010) , “if the null hypothesis of pure noise is rejected by the H2, H3 or H4 tests , this then signifies the presence of structure in the data that cannot be modeled by ARCH or GARCH or stochastic volatility models that assume a pure noise input “.

3. EMPIRICAL RESULTS

The empirical analysis is based on daily closing quotations of SM indexes during the period from March 30, 2010 to Feb.6, 2013, taken from Blumberg. The first part of the analysis is done by Eviews 7.1 software while the second part, which is related to the estimation using higher order cumulants, is done by using MATLAB and by using subroutines files written by the authors of this article.

The analysis consists of two steps. In the first step the best dynamic regression model is found. The Swiss Market Index returns are explained in terms of the international stock market index returns which Granger cause its change, to include DJIA, SP500, NASDAQ ,DAX, FTSE100, NIKKEY225, BSE as well as its own trading volume.

After finding the regression trend model, de-trended time series $\{e_t\}$ is calculated and squared. The squared $\{e_t\}$ is then used to produce both ARMA-GARCH and HOC-ARMA volatility model. The analysis begins with a statistical description of the distributions for the daily SM returns. The results appear in Table 1 (Appendix). The statistics reported confirm that impression: the sample skewnesses are near 0 for both series, but the sample kurtoses are well above the normal value of 3. Jarque-Bera and curtsies values show that in all cases return distribution is leptokurtic.

Proceeding to the Granger causality test to identify what SM indices Granger cause the Swiss Market Index (SMI), surprisingly it is found that the 1-day lagged DAX returns are the only influential for the SM index returns, as presented in Table 2a. However the coefficient of determination of the corresponding regression model is rather small, 5.48 %. The parameter estimates and their standard deviations are given in Table 2b.

De-trended SMI returns $\{\varepsilon_t\}$ are squared to get E2SMI series, as labeled in Eviews. Its graph is presented in Figure 1. It has the skewness of 7.37, kurtosis of 79.45 and Jarque Bera index of 1851.37, as it is presented in Table 3. This description demonstrates a strong departure from Gaussianity and indicates the need to use higher order cumulants for a model building.

The GARCH ARMA parameter estimates based on Eviews 7.1 software are given in Table 4a, while the corresponding parameter estimates based on higher order cumulants, ARMA-HOC estimates, are presented in Table 4b.

From parameter estimates, proceeding with the creation of standard 1-day-ahead forecasts, resulted in the conditional volatilities for the SMI index, as presented in Figures 2a and 2b. As it can be seen, those figures clearly reveal divergence between GARCH-ARMA and ARMA_HOC volatility series. It is important to underline that the SMI volatility estimated using both the second and the fourth order cumulants appears to be higher than one based on the second order cumulants solely. Thus classical GARCH model underestimates the SM volatility and consequently the stock market risk.

Going on, the residuals from each model are calculated by using equation (7). Ultimately, the fourth order cumulants of the squared detrended returns are calculated by using equation (9). Cumulants are presented in Table 5. They are marked as: cum4e2smi, cum4ni-smi and cum4hocni-smi respectively.

For easiness of visual comparison, Figure 3 shows the fourth order diagonal cumulants of those three series. It demonstrates that HOC-ARMA innovations $\{\nu\}$have the forth order cumulant function close to 0 for all lags: l=1, 2…25.

In addition, the tree-dimensional fourth order cumulants for all lags are presented in Figures 4a and 4b. Those figures confirm that the ARMA-HOC model has largely eliminated the interdependence of non-Gaussian residuals, while the GARCH-ARMA model appeared to be insufficient to diminish the fourth order cumulant function, including the excess kurtosis C3(0,0,0) of the Swiss market squared de-trended returns.

Not surprisingly the Hinich statistics (eq. 16), which is based on the fourth order cumulants, for ARMA-GARCH model residuals is 113.102 while the corresponding statistics for the ARMA-HOC cumulants is 12.320. The critical value χ^2_{crit} is 67.50 for both residual cumulants with the level of significance of 5%. This discrepancy demonstrates that ARMA-GRACH parameter estimation method, based on the second order statistics, does not produce independent residuals while ARMA-HOC parameter estimation method does produce independent residuals.

4. CONCLUSION

At the present time, the state of art of stock market volatility forecasting improvement leads either to a new analytical form of the GARCH class models or to the discovery of a new model testing methods which would include dynamics of the unknown non Gaussian probability density function of squared residuals and their innovations, characterized by excess kurtosis.

This paper suggests reorientation of the field around non Gaussian Time Series Analysis main principals. It triggers the paradigm shift by suggesting usage of the third and the fourth cumulant functions, as the sufficient statistics for the model parameter estimation prior to model testing. The new estimation method, which is based on higher order cumulant functions, is carefully described.

The empirical analysis is based on daily stock market index data. The time horizon includes Oct 4, 2007 and Oct.12, 2010. The Swiss Market Index returns (SMI) are Granger tested for the causality with respect to DJIA, SP500, NASDAQ, DAX and FTSE100 index. Thus, the model building is accomplished in two steps. In the first step, a dynamic regression model is developed in order to capture international stock market co-movements. In the second step, which is essential for this paper, the squared de-trended SMI returns are described by using both HOC-ARMA model and GARCH-ARMA model. ARMA –HOC parameter estimation is performed by using the second, the third and the fourth-order cumulants. ARMA-GARCH parameter estimation is performed by using GED based method available in Eviews. Innovations produced by both methods are tested by using the Hinich (H_N) statistics. In terms of Time Series Analysis and digital whitening of SM squared returns, it was demonstrated that ARMA-HOC parameter estimation method performs better in producing independent (or white) residuals.

While the evidence from the International Stock Markets shows that the ARMA parameter estimation, which is based on the higher order cumulant functions, successfully captures non-Gaussian properties of the real stock market returns, a future research goal is yet to be accomplished: it remains to be investigated if HOC realized volatility, based on high frequency prices, constitutes a better proxy for the volatility forecast or whether dynamic HOC-ARMA estimates of daily squared returns perform more efficiently. Nevertheless, the statistical and computational efficiency of the HOC based ARMA parameter estimation method is to be investigated further.

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Appendix : Tables and Figures

Table 1: Statistical description of the Stock Market Returns

	DAX	DJIA	FTSE100	NIKKEY100	NSQ	SMI	SP500
Mean	0.0123	0.0166	0.0069	0.0084	0.0221	0.0048	0.0168
Median	0.0230	0.0224	0.0073	0.0184	0.0370	0.0114	0.0224
Maximum	2.2628	1.8038	2.1855	2.3983	2.1345	2.1293	2.0115
Minimum	-2.6034	-2.4781	-2.0756	-4.8439	-2.7383	-1.8426	-2.9948
Std. Dev.	0.6231	0.4569	0.4872	0.5690	0.5420	0.4412	0.5055
Skewness	-0.1272	-0.3972	-0.1387	-0.9202	-0.2469	-0.2245	-0.4135
Kurtosis	5.2594	6.5391	4.8959	10.6873	5.5406	6.5856	6.8803
Jarque-Bera	158.3148	402.9020	112.4387	1913.4690	205.1419	399.8952	482.0638

Table 2a: Granger test results

	F	Prob.
DAX does not Granger Cause SMI	10.557	0.000
DJIA does not Granger Cause SMI	0.164	0.849
FTSE100 does not Granger Cause SMI	1.286	0.277
NIKKEY100 does not Granger Cause SMI	0.533	0.587
NSQ does not Granger Cause SMI	0.177	0.838
SMIVOL does not Granger Cause SMI	0.475	0.622
SP500 does not Granger Cause SMI	0.253	0.777

Table 2b: SMI regression estimâtes

Variable	Coefficient	St error	t-Stat	Prob.
DAX	0.1277	0.0256	4.9856	0.000
DAX(-1)	0.0553	0.0257	2.1535	0.001
DAX(-2)	0.0864	0.0256	3.3725	0.000
R-squared	0.054838			

Table 3: Descriptive Statistics –all SM squared de-trended returns

	R2BSE	R2DAX	R2DJIA	R2FTSE	R2NIKKEY100	R2NSQ	E2SMI	R2SP
Mean	2.135	0.388	0.209	0.237	0.323	0.294	0.184	0.255
Median	0.075	0.087	0.046	0.063	0.122	0.074	0.046	0.052
Maximum	862.851	6.778	6.141	4.776	23.463	7.499	6.848	8.969
Minimum	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Std. Dev.	37.858	0.800	0.489	0.468	1.004	0.623	0.475	0.616
Skewness	20.324	4.055	5.669	4.545	17.598	5.132	7.373	6.540
Kurtosis	427.651	22.970	48.490	32.185	389.909	40.972	79.448	68.566
Jarque-Bera	5573147	14227	67309	28615	4622442	47383	185137	136894

Table 4a. ARMA-GARCH parameter estimates

	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	MA(4)
Coeff.	0.464	-0.414	0.914	-0.107	0.405	-0.789	-0.202
St.error	0.018	0.020	0.017	0.041	0.026	0.027	0.040
t-Stat	25.638	-21.101	52.969	-2.628	15.397	-29.603	-5.074

Table 4b. ARMA-HOC Parameter estimates

	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	MA(4)
Coeff.	0.040	0.065	0.127	0.087	0.784	0.357	-0.010
St.error	0.019	0.029	0.019	0.039	0.127	0.187	0.004
t-Stat	2.121	2.243	6.616	2.230	6.180	1.910	-7.285

Table 5.The fourth order diagonal cumulants

Lags	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Cum4e2smi	3.91	1.23	0.21	0.20	0.19	0.68	0.12	0.33	0.13	0.02	0.03	0.03	0.24	0.31
Cum4smini	1.79	0.32	0.19	0.19	0.08	0.39	0.20	0.27	0.04	0.02	0.04	0.06	0.15	0.19
Cum4smihocni	1.83	0.38	0.15	0.17	0.04	0.20	0.13	0.30	0.01	0.00	0.00	0.03	0.01	0.01

Figures:

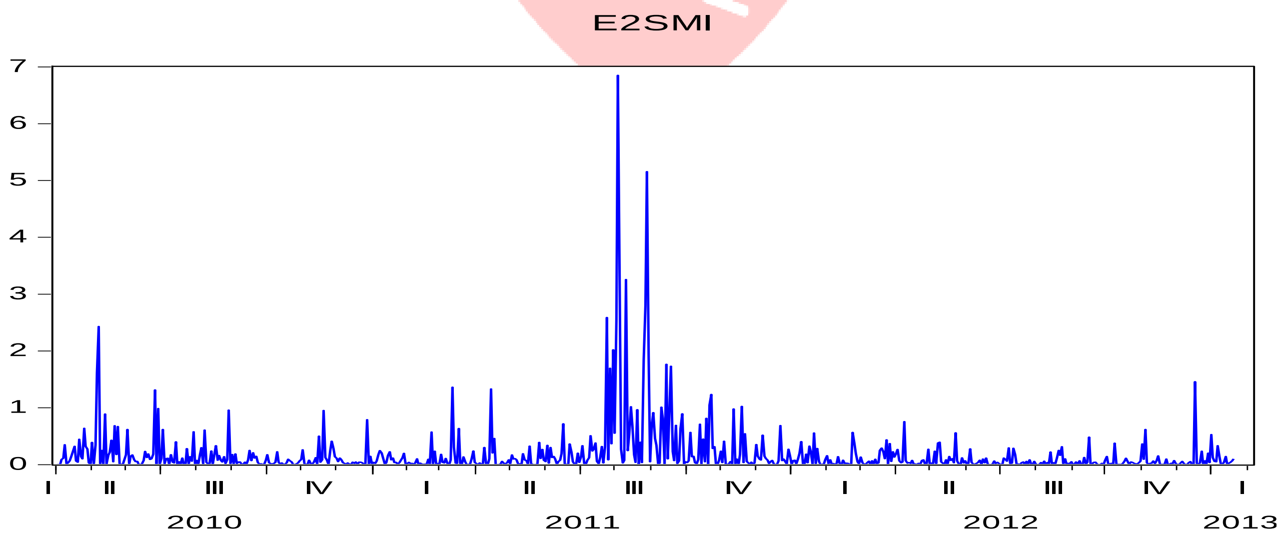
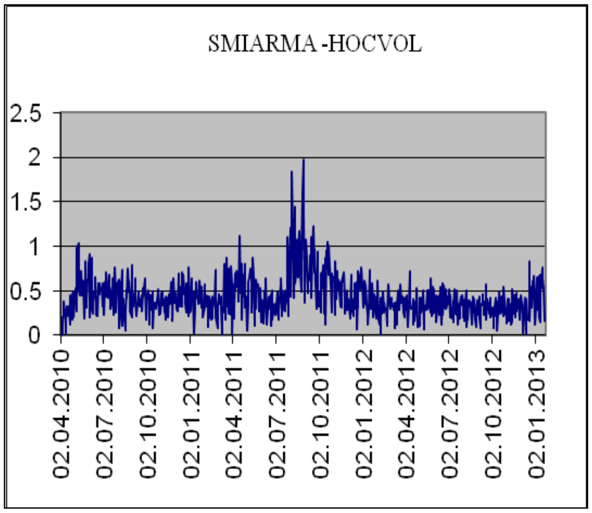


Figure 1: De-trended Swiss Market Index -squared returns



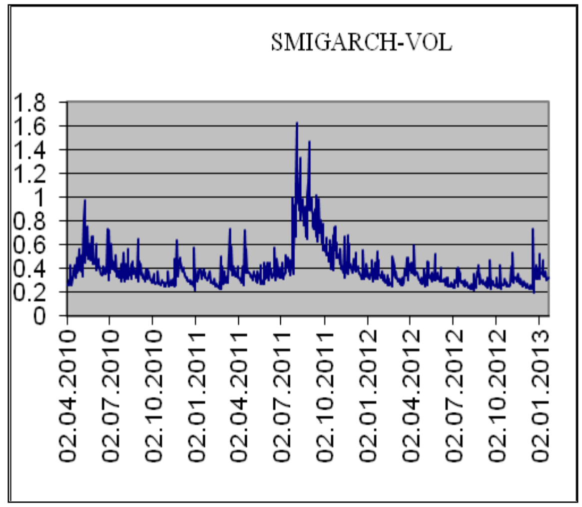


Figure 2a: Swiss Market index –GARCH volatility Figure 2b: Swiss Market Index: HOC Volatility

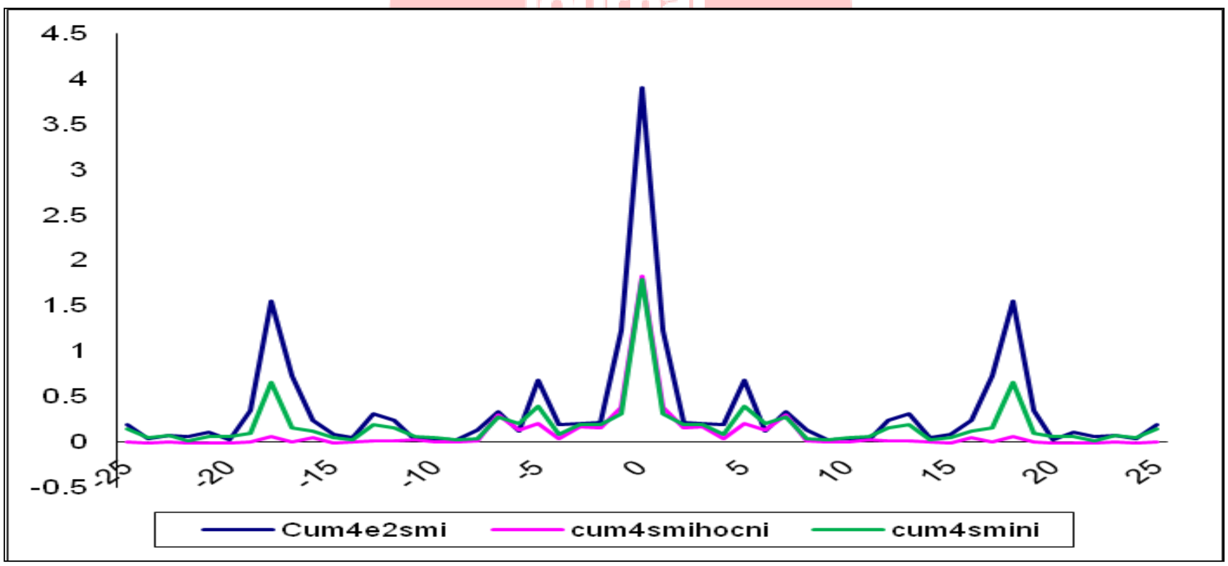


Figure 3: Fourth order cumulants of the squared de-trended SMI returns and its ARMA GARCH and ARMA-HOC residuals

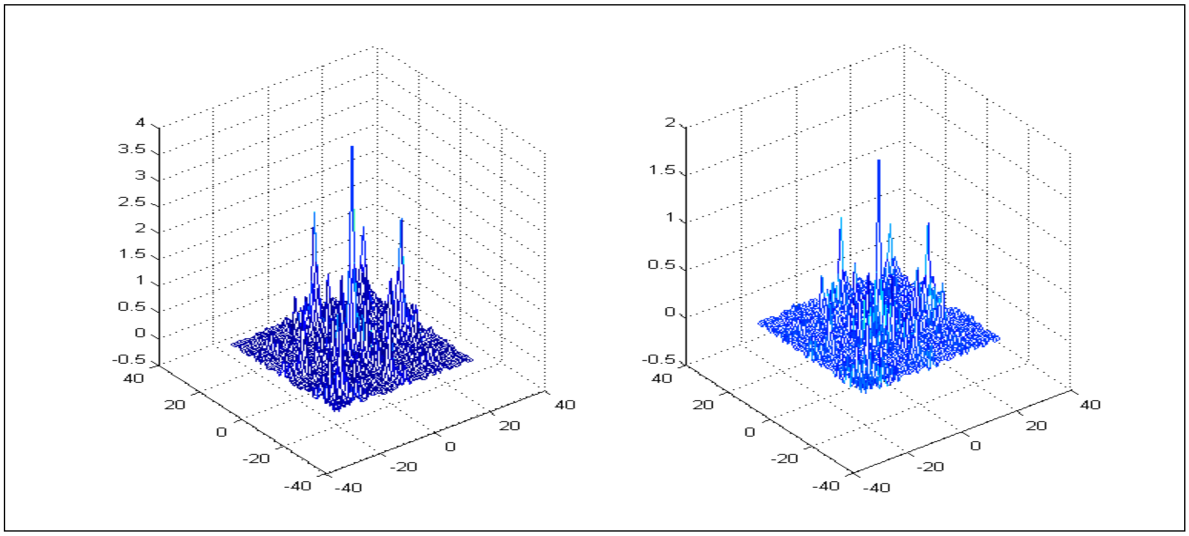


Figure 4a: The fourth-order cumulants of the Swiss Market Index squared de trended returns and its ARMA-GARCH residuals.

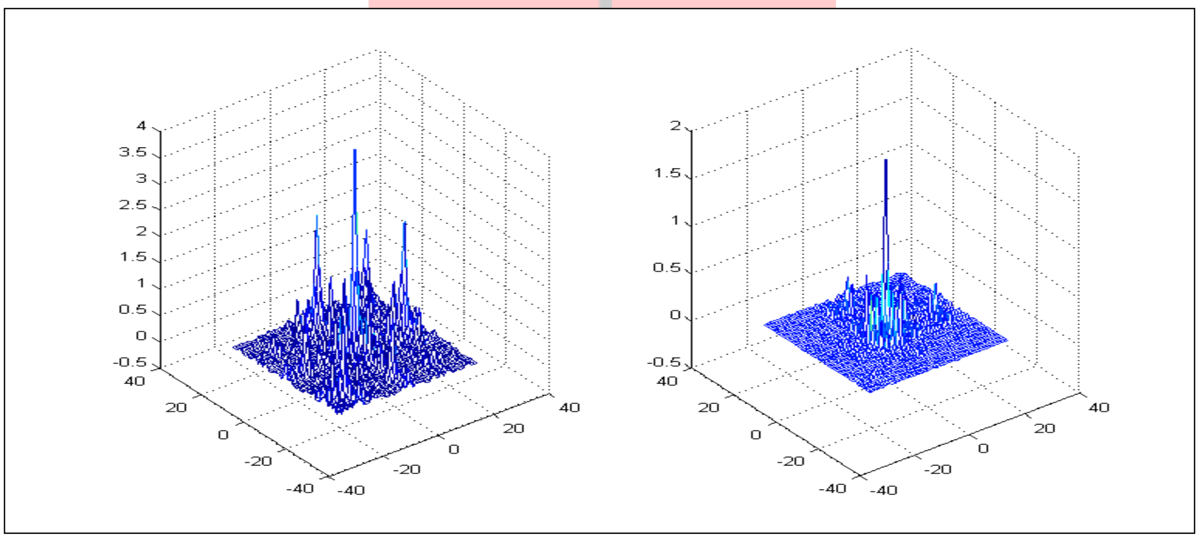


Figure 4b: The fourth-order cumulants of the Swiss Market Index squared de-trended returns and its HOC -ARMA residuals.