An introduction to Benford's Law: the Case of MiMedx

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ABTRACT

This paper aims to introduce undergraduate business students to Benford's Law through an exploration of MiMedx's alleged financial statement fraud from 2012 to 2017. After its introductory section, the paper provides background on the alleged fraud and on Benford's Law. Then, it presents a series of three exercises focused on Benford's Law of second digits. The first exercise is a longitudinal analysis of line items on MiMedx's financial statements that appear to have been the most vulnerable to fraudulent misstatement. The second exercise allows students to conduct a more in-depth, cross-sectional analysis of MiMedx's 2015 financial statements, with 2015 having been a year when financial statement fraud may have been especially pronounced. Finally, the third exercise reminds students to consider possible shortcomings of their analysis.

Keywords: forensic accounting, financial statement fraud, Benford's Law, channel stuffing, inventory fraud, business education

Acknowledgment: The authors would like to thank the Virginia Military Institute's Center for Undergraduate Research (VCUR) for providing a summer research grant for this project.

INTRODUCTION

Regulatory efforts to prevent and detect financial statement fraud[1] have been historically problematic. Despite the evolution and the proliferation of national and international financial regulators, financial statement fraud has remained endemic. An endless sea of frauds has spanned over the last century from the McKesson and Robbins, Inc. scandal in the United States in 1938 (MacDonald, 1999; McLeod, 2015) to the recent international Luckin Coffee scandal (SEC, 2020).

The initial detection of financial statement fraud has often sprung from outside of mainstream regulatory efforts. Detection commonly occurs through the work of hedge funds, as in the case of the Luckin Coffee scandal (Baskett, 2020). It also frequently occurs through the efforts of investigative journalists, as in the cases of the MiMedx scandal (Morgenson, Walker, & Grant, 2018) and the WireCard scandal (McCrum & Palma, 2019). Frauds detected in this manner are likely to become large enough that some investors have (or will) suffer from significant and unrecoverable losses. These frauds also often go on for protracted periods of time.[2] Consequently, the authors argue that early fraud detection within regulatory channels via data analytics may be a productive aim.

Today's students will be tomorrow's regulators. Accordingly, this paper provides three classroom exercises to introduce undergraduate business students to Benford's Law (Benford, 1938; Nigrini & Wells, 2012) as an analytical approach for identifying potential fraud. These exercises encourage students to consider whether Benford's Law might have facilitated earlier detection of the MiMedx scandal.[3] They also help students consider possible shortcomings of using Benford's Law to identify financial statement fraud.

The remainder of this paper is organized as follows. The next section provides background information on the alleged MiMedx fraud and on Benford's Law. The following section presents three exercises designed to help students learn to apply Benford's law. Suggested solutions are provided in the paper's Teaching Note.

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[1]The Association of Certified Fraud Examiners (ACFE) (2020) defines financial statement fraud as "the deliberate misrepresentation of the financial condition of an enterprise accomplished through the intentional misstatement or omission of amounts or disclosures in financial statements to deceive the financial statement users" (*Fraud Examiner's Manual*, Section 1, Financial Transactions and Fraud Schemes).

[2]For example, in terms of the recent scandals noted in this introduction, allegations of financial impropriety were made about Wirecard at least as far back as 2008 (McCrum, 2020). Yet, the company did not become insolvent until June 2020 after $2 billion in cash was reported missing. Luckin Coffee (Baskett, 2020) appeared to have been involved in fraudulent reporting since at least when it became a public company in May 2019. However, the company went on to being valued as high as $12 billion before being delisted in June 2020. Although indications of fraud at MiMedx did not begin to emerge until 2018, the SEC eventually alleged that the financial statement fraud began in 2013 and continued through the third quarter of 2017 (*Securities and Exchange Commission [SEC] v. MiMedx Group, Inc., Petit, Taylor, & Senken*, 2019).

[3]The authors selected the MiMedx scandal due to the corporation's relatively straightforward financial statements, which makes a discussion of financial statement fraud more accessible for undergraduate students.

BACKGROUND

Synopsis of MiMedx Fraud

Until recently, MiMedx—a pioneer in developing products for wound care made from human placentas—was a darling of the investment world. Reporting revenue growth that exceeded 50% each year from 2012 to 2016, MiMedx was purportedly at one time the fifth fastest-growing public company in the United States according to *Fortune* (100 Fastest-Growing Companies, 2017; Morgenson et al., 2018). However, largely through *Wall Street Journal's* reporting efforts, it became apparent in 2018 that MiMedx was probably falsifying its financial statements to achieve its staggering growth (Morgenson, 2018a; 2018b; Morgenson et al., 2018). In particular, MiMedx appears to have inflated its sales through channel stuffing,[4] through early recognition of inventory held on consignment as revenue,[5] and through false price disclosures to governmental agencies.

The resulting complaint (*SEC v. MiMedx Group, Inc., et al., 2019*) describes MiMedx as being engaged in channel stuffing through side-agreements with its five largest distributors at various points from 2013 to the third quarter of 2017. The complaint indicates that sales for the last three quarters of 2015 were especially likely to have been fraudulent—with revenues for each of these quarters being overstated by 6% to 14%.

Employees attempted to alert MiMedx about record-keeping discrepancies that suggested inventory held on consignment had been recorded as revenue prematurely instead of being reported as an asset (i.e., inventory) on its balance sheet (Morgenson et al., 2018). Although the company stated that it encouraged internal whistleblowing and did not engage in retaliation, some employees were terminated shortly after their internal whistleblowing attempts. MiMedx provided alternative explanations for these terminations and lawsuits ensued.

MiMedx also allegedly forced the Veterans Affairs Administration and the Department of Defense to buy more expensive products than it offered to private hospitals and doctors (Morgenson, 2018b). Analysis demonstrated that the government was one of the company's major customers, making this overpricing significant. For example, revenue from the federal government comprised more than a quarter of the company's annual revenue in 2015.

In December 2018, Ernst & Young—recently engaged as MiMedx's auditor—resigned, declaring that it was unable to complete its audit (Chin, 2018; Edwards, 2018). "Ernst & Young determined that . . . internal controls needed to produce accurate financial statements 'do not exist' "(Edwards, 2018, para 2.). MiMedx announced that its financial filings as far back as 2012 were likely to be unreliable (Chin, 2018; Grant & Morgenson, 2018).

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[4]Channel stuffing is used to inflate revenue by recording accounts receivable in excess of legitimate customer purchases. For example, if a company debits accounts receivable to record $1 million in fictitious accounts receivable at the end of the year, then sales revenue will also be credited (or inflated) by $1 million.

[5]Inventory on consignment is recorded as sales (or revenue) only when sold. In this case, hospitals held MiMedx's products for use on patients; thus, sales should have been recorded only when the products were used for medical care. MiMedx often allegedly recorded sales prematurely to meet high-pressure, earnings forecasts.

Unable to produce its financial statements for 2017, MiMedx was delisted from the NASDAQ (Grant & Morgenson, 2018). The SEC filed a complaint against the company and three of its former executives in November 2019 (*SEC v. MiMedx Group, Inc., et al., 2019*). MiMedx settled civil fraud charges for $1.5 million with the SEC without admitting guilt (SEC, 2019). It also paid $6.5 million in April 2020 to settle charges that it violated the False Claims Act by excluding less expensive products from the pricelists that it had provided to the Veteran's Affairs Administration (Department of Justice, 2020).

What is Benford's Law?

Originally discovered by Simon Newcomb in 1881 (Collins, 2017), Benford Law predicts the expected frequency of digits in naturally occurring numbers (Benford, 1938). This approach became more widely known following an article in *Wall Street Journal* in 1995, which describes Mark Nigrini's use of Benford's Law to detect fraud in tax returns and checks (Berton, 1995; Nigrini, 2012).

Benford's best-know prediction is often called the first-digit test, where the first digit is the left-most digit in a number (Collins, 2017; Nigrini, 2012). As Table 1 (Appendix) shows, a "1" is expected to be the first digit in a naturally occurring set of numbers 30% of the time. A "2" is expected to be the first digit 17.6% of the time, and so forth. Observed frequencies of digits in empirical datasets can be evaluated graphically for their conformity with Benford's Curve, which is a graphical representation of Table 1 (Figure 1, Appendix). Nigrini (2012) discusses the use of statistical tests to evaluate conformity with Benford's Law such a *z* score for smaller datasets and a chi-squared test and a Mean Absolute Deviation (MAD) test for larger datasets.

A violation of the first-digit test does not necessarily suggest financial statement fraud. Rather, Nigrini (2012) explains that the first-digit test "is usually of too high a level to be of too much use [except with small datasets, with 300 data points being suggested as being a small dataset]" (p. 74). Instead, Nigrini (2012) indicates that the second-digits tests[6] "works very well to detect *biases* in data… when people aim for specific numbers or number ranges to circumvent actual or perceived internal control thresholds" (p. 75-76). The predicted frequency of the second digit is actually predicated on the first digit closely conforming with Benford's predictions (Nigrini, 2012).[7]

Although potentially promising for detecting financial statement fraud, Benford's Law has several shortcomings. Collins (2012) identifies some of these as follow:

- Benford's Law is not probative. It identifies only the possibility of fraud.
- Although Benford's Law is potentially useful in small datasets, its use in larger datasets is generally recommended; and
- Benford's Law applies to naturally occurring numbers.

Nigrini (2012) suggests that additional problems with the application of Benford's Law include improper data screening (e.g., failing to screen out duplicates) and a lack of definitive benchmarks for statistical tests that involve large datasets.

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[6]Nigrini (2012) also points out that in other circumstances, the second-digit test may be "too high level to be of much use" (p. 75).
[7]The authors introduce the second digit-test in this exercise. However, students should be aware that several other variations of Benford's law exist (Nigrini, 2012).

THE EXERCISES

The authors suggest using one of the following as a source of financial data for this exercise: (1) the SEC's EDGAR System at https://www.sec.gov/edgar/ or (2) a propriety database (as such S&P Capital IQ). Students should be sure to use data from MiMedx's original 10-Ks, rather than its amended ones.

Exercise 1. Evaluate key, longitudinal line items on the financial statements for possible indications of fraud at MiMedx.[8]

1. Input the following into an Excel spreadsheet from MiMedx's original, annual financial statements from 2012 to 2016 found in Item 8 of the company's 10-Ks:

   Group A. Revenue, net income, operating income, and earnings per share (basic)
   Group B. Accounts receivable, net
   Group C. Finished goods and COGS

2. Apply the second-digit rule of Benford's Law to search for possible biases that might be associated with fraud (Nigrini, 2012).

   a. Locate any duplicate occurrences of second-digits for each line-item in the financial statements from 2012 to 2016, where the second digit is the second digit from the left (not the right).

   b. Use the binomial probability distribution function in Excel (=BINOMDIST(numbers_s, trials, probability_s, False)[9] to evaluate the probability that this pattern of repetitions would occur naturally.

   To illustrate how to input arguments into the probability distribution function, suppose that the second digit of accounts receivable is 3 for four of the five years from 2012 to 2016. The BINOMDIST function will be =BINOMDIST(4, 5, 0.10433, False) or .00053. The first argument in the function is the number of times the digit 3 is repeated; the second argument is the number of years (i.e., 5 years from 2012 to 2016); the third argument is the expected frequency of a 3 occurring naturally as a second digit from Table 1. The result of .00053 is interpreted as there being 53 out of 100,000 chances that a 3 would appear four times as the second-digit in a naturally occurring in a series of five numbers.

Explanatory Notes:

- Group A allows for a longitudinal assessment of revenue-related items under Benford's Law. Group B allows for a longitudinal assessment of evidence of channel stuffing. Group C allows for a longitudinal assessment that might detect improper recording of inventory on consignment, after further evaluation.
- The years 2012 to 2016 are suggested in this exercise because MiMedx stated that its financial filings as far back as 2012 might be unreliable, with 2016 being the cut off

[8] The authors adapted Nigrini (2012)'s and Nigrini (2005)'s analysis of Enron for this question.
[9] "False" indicates that the function is a probability mass function.

year because MiMedx could not produce its 2017 10-K following the resignation of E&Y (Chin, 2018; Grant & Morgenson, 2018).

Exercise 2. Evaluate MiMedx's full set of audited financial statement data for 2015.

1. Obtain and screen the data. Input all the numbers in MiMedx's original 10-K filing for 2015 (for Item 8 in the 10-K) into Excel. Cull the following from the dataset (Nigrini, 2012):
   - Meaningless numbers (such as page numbers, dates etc.);
   - Transformations of numbers such as subtotals, totals, and foreign exchange translations;
   - Percentages and duplicates (i.e., net income on the statement of cash flows and net income on the income statement);
   - Zero (when expressed as an entire number, but not when expressed as a digit in the number); and
   - Numbers that do not have at least two digits.
   - In the analysis presented, the authors evaluated only positive line items.

2. Conduct the first-digit test in Excel. [10] Recall that conformity with this test is needed to evaluate Benford's Law of second digits. An example of a mock dataset and Excel formulas is provided in Figure 2 (Appendix).
   - Create a column that extracts the first digit of each number in the dataset, using the Left function (Columns C, Figure 2, Appendix).
   - Create a column that counts the number of times that each digit from 1 to 9 appears as the first digit using the Countif function (Column E, Figure 2, Appendix).
   - In the next column, express the actual frequency of each digit as a percent (Column F, Figure 2, Appendix).
   - Input the expected frequencies based on Benford's Law from Table 1 in the next column (Column G, Figure 2, Appendix).
   - Calculate the *z* scores[11] to compare the actual and predicted frequencies (Column H Figure 2, Appendix). The calculation in Figure 2 (Appendix) labeled "absolute difference," "1/2N," "*z* score numerator," and "*z* score denominator" are used in the calculation of the *z* score. If the *z* score is 1.96 or higher, the observed frequency and

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[10] Rather than having students create the Excel files, instructors may prefer to create the spreadsheets for students or use template called NigriniCycle.xlsx available at https://www.dropbox.com/s/6g1k7ffg3pvamwu/NigriniCycle.xlsx?dl=0

[11] The formula for Z-score (Nigrini, 2012) is (Absolute value(Observed Percentage – Expected Percentage) – (1/2N))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) > (1/2N), where N is the number of data items or (Absolute value(Observed Percentage – Expected Percentage))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) < (1/2N),), where N is the number of data items.

the predicted frequency are significantly different for an alpha of .05 (or a 95% significance level).[12]

- The observed frequencies can also be evaluated graphically in term of their similarity to Figure 1.
- If the data conform to the first-digit test, then proceed to the next step. If not, further analysis may not be warranted because, as explained in the background section, the distribution for the second-digit test is contingent of close conformity with the first-digit test (Nigrini, 2012).

3. Conduct the second-digit test in Excel. An example of a mock dataset and Excel formulas is provided in Figure 3 (Appendix).
    - Create a column that extracts the second digit of each number in the dataset, using the Mid function (Column C, Figure 3, Appendix).
    - Create a column that counts the number of times that each digit from 0 to 9 appears as the first digit using the Countif function (Column E, Figure 3, Appendix).
    - In the next column, express the observed frequency of each digit as a percent (Column F, Figure 3, Appendix).
    - Input the expected frequencies based on Benford's Law for the second digit from Table 1 in the next column (Column G, Figure 3, Appendix).
    - Calculate z scores[13] to compare the observed and predicted frequencies (Columns H Figure 3, Appendix). The calculation in Figure 3 (Appendix) labeled "absolute difference," "1/2N," "*z* score numerator," and "*z* score denominator" are used in the calculation of the *z* score If the z score is 1.96 or higher, the observed frequency and the predicted frequency in Benford's Law are significantly different for an alpha of .05 (or for a 95% significance level).
    - The observed frequencies can also be evaluated graphically in terms of their similarity to Figure 1.

Explanatory note:
The authors selected 2015 for cross-sectional analysis, due to indications in the SEC complaint (*SEC v. MiMedx Group, Inc., et al., 2019*) that it may have been a particularly fraud-ridden year.

Exercise 3. What are some of the limitations of Benford's Law as it applies to Exercise 1 and Exercise 2?

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[12]The authors recommend the use of a *z* score for this dataset due its small sample size. Nigrini (2012) discusses the use other statistical tests such a chi-squared test and a Mean Absolute Deviation (MAD) test for larger datasets.

[13]The formula for Z-score (Nigrini, 2012) is (Absolute value(Observed Percentage – Expected Percentage) – (1/2N))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) > (1/2N), where N is the number of data items or (Absolute value(Observed Percentage – Expected Percentage))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) < (1/2N),), where N is the number of data items.

TEACHING NOTE (SOLUTIONS)

Exercise 1.Evaluate key, longitudinal line items on the financial statements for possible indications of fraud at MiMedx.

Panel 4 of Table 2 (Appendix) suggests that the probability of the duplications observed is relatively low. This is especially the case for the observation of the digit 6 three times in accounts receivable and the digit 0 three times in EPS.

Exercise 2. Evaluate MiMedx's full set of financial statement date for 2015.

Co-author 1 in consultation with co-author 2 screened the data as described in the dataset; 71 observations were available after this step for data analysis. As shown in Table 3 (Appendix), the data appeared to be in conformity with the first-digit test, given that none of the *z* scores exceeded 1.96. This close conformity is a necessary condition for moving forward to the second-digit test. However, the second-digit tested yielded a *z* score of 1.99 for the comparison between the observed frequency of 6s compared to the expected frequency. Thus, the evidence suggests possible bias in MiMedx's financial data. It is interesting to note that this possible bias in the pattern of 6s is consistent between Exercise 1 and Exercise 2.

Exercise 3. What are some of the limitations of Benford's Law as it applies to Exercise 1 and Exercise 2?

The primary limitation of the analysis is that any irregularities detected are not proof of fraud (Collin, 2017). They suggest only the possibility of fraud, with more in-depth investigation being needed. Additionally, students should consider whether MiMedx's financial data represents a set of naturally occurring numbers, which is a prerequisite for Benford's Law. MiMedx's pricing schedules should be examined to determine whether they could account for the results of Exercise 1 and 2. It seems unlikely that this would be the case. Nigrini (2012) concludes that financial statement data does generally adhere to Benford's Law. In addition, it seems unlikely that the second digit would be impacted by MiMedx's pricing schedules. However, further examination of this issue is needed. Finally, students should consider that this application of Benford's Law was based on a relatively small dataset with Benford's Law generally being thought to be most appropriate for large datasets (Collins, 2017).

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APPENDIX

Tables

Table 1: Predicted frequency of digits in naturally occurring numbers per Benford's Law

Position in number (with the left-most digit being the first position)

Digit	1	2	3	4
0	n/a	.11968	.10178	.10018
1	.30103	.11389	.10138	.10014
2	.17609	.10882	.10097	.10010
3	.12494	.10433	.10057	.10006
4	.09691	.10031	.10018	.10002
5	.07918	.09668	.09979	.09998
6	.06695	.09337	.09940	.09994
7	.05799	.09035	.09902	.09990
8	.05115	.08757	.09864	.09986
9	.04578	.08500	.09827	.09982

Source: Adapted from Table 1.2 Nigrini (2012).

Table 2: Solution to Exercise 1—probabilities of observed duplications in second digit occurring in a dataset of natural numbers

	Years				
	2012	**2013**	**2014**	**2015**	**2016**
Panel 1. Longitudinal analysis of second digits for income items					
Total Revenue (in thousands)	27,053.8	59,180.7	118,223.0	187,296.0	245,015.0
Net Income (in thousands)	(7,662.4)	(4,111.9)	6,220.0	29,446.0	11,974.0
Operating Income (in thousands)	(1,989.8)	(2,270.5)	7,100.0	24,364.0	21,127.0
Earnings per Share (EPS), basic	(0.09)	(0.04)	0.06	0.28	0.11
Panel 2. Longitudinal analysis of second digits for accounts receivable (channel stuffing)					
Accounts Receivable (A/R), net (in thousands)	7,653.6	16,092.8	26,672.0	53,755.0	67,151.0
Panel 3. Longitudinal analysis of second digits for accounts possibly related to consignment inventory					
Finished goods inventory (in thousands)	1,349.1	1,048.8	1,986.0	3,405.0	10,817.0
Cost of Goods Sold (COGS) (in thousands)	5,188.4	9,328.1	12,665.0	20,202.0	30,814.0
Panel 4. Probabilities*of repeated digits occurring by chance in a naturally occuring set of numbers					
	as fraction	as decimal			
probability of second digit being 1 twice in net income	62/687	0.090247			
probability of second digit being 1 twice in operating income	62/687	0.090247			
probability of second digit being 0 three times in EPS, basic	11/828	0.013285			
probability of second digit being 6 three times in A/R, net	2/299	0.006691			
probability of second digit being 0 twice in finished goods inventory	77/788	0.097716			
probability of second digit being 0 twice in COGS	77/788	0.097716			

***Probabilities calculated using the binomial probability distribution function =BINOMDIST(numbers_s,trials, probabilities, False)**

Table 3: Solution to Exercise 2— z scores for the first-digit test and the second-digit test ($N = 71$)

Panel 1: First-digit test (results)			
First digit	Actual	Predicted	*z* score
1	.0268	.30103	0.477
2	.141	.17609	0.62
3	.141	.12494	0.23
4	.155	.09691	1.454
5	.07	.07918	0.067
6	.056	.06695	0.132
7	.085	.05799	0.72
8	.056	.05115	0.186
9	.028	.04578	0.432
Panel 2: Second-digit test (results)			
Second digit	Actual	Predicted	z score
0	.183	.11968	1.461
1	.099	.11389	0.028
2	.056	.10882	1.239
3	.07	.10433	0.752
4	.099	.10031	0.037
5	.141	.09668	1.063
6	**.169**	**.09337**	**1.986**
7	.056	.09035	0.803
8	.07	.08757	0.314
9	.056	.08500	0.663

Significant results are bolded.

Figures

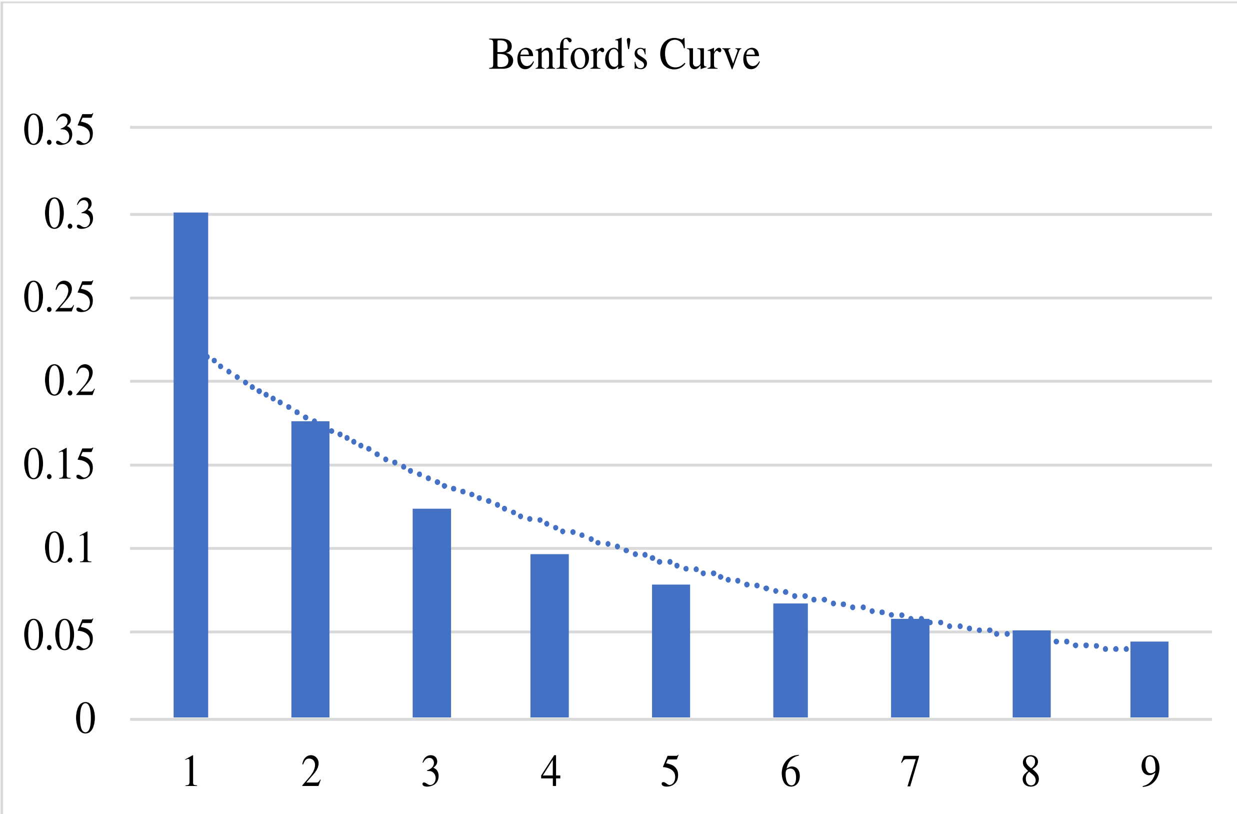

Figure 1. Benford’s Curve is a graph of the predicted frequencies for the first-digit provided in Table 1. Empirical datasets can be evaluated visually for their conformity with Benford’s Curve.

	A	B	C	D	E	F	G	H
1	**DATA**			**First-digit test**				
2			**first digit**			**actual**	**Benford's**	**absolute**
3			**extracted**	**digit**	**count**	**frequency**	**law**	**difference**
4	Cash	204,500	2	1	0	0	0.30103	0.30103
5	A/R	26,005	2	2	4	0.8	0.17609	0.62391
6	finished goods	20,555	2	3	0	0	0.12494	0.12494
7	equipment	275,000	2	4	0	0	0.09691	0.09691
8	goodwill	50,000	5	5	1	0.2	0.07918	0.12082
9				6	0	0	0.06695	0.06695
10				7	0	0	0.05799	0.05799
11				8	0	0	0.05115	0.05115
12				9	0	0	0.04578	0.04578
13					5			
14						***z*** **score**	***z*** **score**	***z*** **score**
15				**digit**	**1/2N**	**numerator**	**denominator**	
16				1	0.1	0.20103	0.20514	0.97997
17				2	0.1	0.52391	0.17034	**3.07563**
18				3	0.1	0.02494	0.14787	0.16866
19				4	0.1	0.09691	0.13230	0.73249
20				5	0.1	0.02082	0.12076	0.17241
21				6	0.1	0.06695	0.11177	0.59897
22				7	0.1	0.05799	0.10452	0.55480
23				8	0.1	0.05115	0.09852	0.51917
24				9	0.1	0.04578	0.09347	0.48978

Panel 1. Numerical results.

	A	B	C	D	E	F	G	H
1	**DATA**			**First-digit test**				
2			**first digit**			**actual**	**Benford's**	**absolute**
3			**extracted**	**digit**	**count**	**frequency**	**law**	**difference**
4	Cash	204500	=LEFT(B4,1)	1	=COUNTIF($C$4:$C$8,D4)	=E4/$E$13	0.30103	=ABS(F4-G4)
5	A/R	26005	=LEFT(B5,1)	2	=COUNTIF($C$4:$C$8,D5)	=E5/$E$13	0.17609	=ABS(F5-G5)
6	finished goods	20555	=LEFT(B6,1)	3	=COUNTIF($C$4:$C$8,D6)	=E6/$E$13	0.12494	=ABS(F6-G6)
7	equipment	275000	=LEFT(B7,1)	4	=COUNTIF($C$4:$C$8,D7)	=E7/$E$13	0.09691	=ABS(F7-G7)
8	goodwill	50000	=LEFT(B8,1)	5	=COUNTIF($C$4:$C$8,D8)	=E8/$E$13	0.07918	=ABS(F8-G8)
9				6	=COUNTIF($C$4:$C$8,D9)	=E9/$E$13	0.06695	=ABS(F9-G9)
10				7	=COUNTIF($C$4:$C$8,D10)	=E10/$E$13	0.05799	=ABS(F10-G10)
11				8	=COUNTIF($C$4:$C$8,D11)	=E11/$E$13	0.05115	=ABS(F11-G11)
12				9	=COUNTIF($C$4:$C$8,D12)	=E12/$E$13	0.04578	=ABS(F12-G12)
13					=SUM(E4:E12)			
14						***z*** **score**	***z*** **score**	***z*** **score**
15				**digit**	**1/2N**	**numerator**	**denominator**	
16				1	=1/(2*$E$13)	=IF(E16<H4,H4-E16,H4)	=SQRT(G4*(1-G4)/$E$13)	=F16/G16
17				2	=1/(2*$E$13)	=IF(E17<H5,H5-E17,H5)	=SQRT(G5*(1-G5)/$E$13)	**=F17/G17**
18				3	=1/(2*$E$13)	=IF(E18<H6,H6-E18,H6)	=SQRT(G6*(1-G6)/$E$13)	=F18/G18
19				4	=1/(2*$E$13)	=IF(E19<H7,H7-E19,H7)	=SQRT(G7*(1-G7)/$E$13)	=F19/G19
20				5	=1/(2*$E$13)	=IF(E20<H8,H8-E20,H8)	=SQRT(G8*(1-G8)/$E$13)	=F20/G20
21				6	=1/(2*$E$13)	=IF(E21<H9,H9-E21,H9)	=SQRT(G9*(1-G9)/$E$13)	=F21/G21
22				7	=1/(2*$E$13)	=IF(E22<H10,H10-E22,H10)	=SQRT(G10*(1-G10)/$E$13)	=F22/G22
23				8	=1/(2*$E$13)	=IF(E23<H11,H11-E23,H11)	=SQRT(G11*(1-G11)/$E$13)	=F23/G23
24				9	=1/(2*$E$13)	=IF(E24<H12,H12-E24,H12)	=SQRT(G12*(1-G12)/$E$13)	=F24/G24

Panel 2. Excel formulas.

Figure 2. Illustration of first-digit test. Illustration of numerical results (panel 1) and Excel formulas[14] (panel 2) for a mock dataset.

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[14] The formula for Z-score (Nigrini, 2012) is (Absolute value(Observed Percentage – Expected Percentage) – (1/2N))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) > (1/2N), where N is the number of data items or (Absolute value(Observed Percentage – Expected Percentage))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) < (1/2N),), where N is the number of data items.

	A	B	C	D	E	F	G	H
30	**DATA**		**Second digit**					
31			**extracted**	**Second-digit test**				
32						**actual**	**Benford's**	**absolute**
33	Cash	204,500	0	**digit**	**count**	**frequency**	**law**	**difference**
34	A/R	26,005	6	0	3	0.6	0.11968	0.48032
35	finished goods	20,555	0	1	0	0	0.11389	0.11389
36	equipment	275,000	7	2	0	0	0.10882	0.10882
37	goodwill	50,000	0	3	0	0	0.10433	0.10433
38				4	0	0	0.10031	0.10031
39				5	0	0	0.09668	0.09668
40				6	1	0.2	0.93370	0.73370
41				7	1	0.2	0.09350	0.10650
42				8	0	0	0.08757	0.08757
43				9	0	0	0.08500	0.08500
44					5			
45						***z* score**	***z* score**	
46				**digit**	**1/2N**	**numerator**	**denominator**	***z* score**
47				0	0.1	0.38032	0.14516	**2.62001**
48				1	0.1	0.01389	0.14207	0.09777
49				2	0.1	0.00882	0.13927	0.06333
50				3	0.1	0.00433	0.13671	0.03167
51				4	0.1	0.00031	0.13435	0.00231
52				5	0.1	0.09668	0.13216	0.73153
53				6	0.1	0.63370	0.11127	**5.69519**
54				7	0.1	0.00650	0.13020	0.04992
55				8	0.1	0.08757	0.12641	0.69273
56				9	0.1	0.08500	0.12472	0.68153

Panel 1. Numerical results.

	A	B	C	D	E	F	G	H
30	**DATA**		**Second digit**					
31			**extracted**	**Second-digit test**				
32						**actual**	**Benford's**	**absolute**
33	Cash	204500	=MID(B33,2,1)	**digit**	**count**	**frequency**	**law**	**difference**
34	A/R	26005	=MID(B34,2,1)	0	=COUNTIF($C$33:$C$37,D34)	=+E34/$E$44	0.11968	=ABS(F34-G34)
35	finished goods	20555	=MID(B35,2,1)	1	=COUNTIF($C$33:$C$37,D35)	=+E35/$E$44	0.11389	=ABS(F35-G35)
36	equipment	275000	=MID(B36,2,1)	2	=COUNTIF($C$33:$C$37,D36)	=+E36/$E$44	0.10882	=ABS(F36-G36)
37	goodwill	50000	=MID(B37,2,1)	3	=COUNTIF($C$33:$C$37,D37)	=+E37/$E$44	0.10433	=ABS(F37-G37)
38				4	=COUNTIF($C$33:$C$37,D38)	=+E38/$E$44	0.10031	=ABS(F38-G38)
39				5	=COUNTIF($C$33:$C$37,D39)	=+E39/$E$44	0.09668	=ABS(F39-G39)
40				6	=COUNTIF($C$33:$C$37,D40)	=+E40/$E$44	0.9337	=ABS(F40-G40)
41				7	=COUNTIF($C$33:$C$37,D41)	=+E41/$E$44	0.0935	=ABS(F41-G41)
42				8	=COUNTIF($C$33:$C$37,D42)	=+E42/$E$44	0.08757	=ABS(F42-G42)
43				9	=COUNTIF($C$33:$C$37,D43)	=+E43/$E$44	0.085	=ABS(F43-G43)
44					=SUM(E34:E43)			
45						***z* score**	***z* score**	
46				**digit**	**1/2N**	**numerator**	**denominator**	***z* score**
47				0	=1/(2*$E$44)	=IF(E47<H34,H34-E47,H34)	=SQRT(G34*(1-G34)/$E$44)	**=F47/G47**
48				1	=1/(2*$E$44)	=IF(E48<H35,H35-E48,H35)	=SQRT(G35*(1-G35)/$E$44)	=F48/G48
49				2	=1/(2*$E$44)	=IF(E49<H36,H36-E49,H36)	=SQRT(G36*(1-G36)/$E$44)	=F49/G49
50				3	=1/(2*$E$44)	=IF(E50<H37,H37-E50,H37)	=SQRT(G37*(1-G37)/$E$44)	=F50/G50
51				4	=1/(2*$E$44)	=IF(E51<H38,H38-E51,H38)	=SQRT(G38*(1-G38)/$E$44)	=F51/G51
52				5	=1/(2*$E$44)	=IF(E52<H39,H39-E52,H39)	=SQRT(G39*(1-G39)/$E$44)	=F52/G52
53				6	=1/(2*$E$44)	=IF(E53<H40,H40-E53,H40)	=SQRT(G40*(1-G40)/$E$44)	**=F53/G53**
54				7	=1/(2*$E$44)	=IF(E54<H41,H41-E54,H41)	=SQRT(G41*(1-G41)/$E$44)	=F54/G54
55				8	=1/(2*$E$44)	=IF(E55<H42,H42-E55,H42)	=SQRT(G42*(1-G42)/$E$44)	=F55/G55
56				9	=1/(2*$E$44)	=IF(E56<H43,H43-E56,H43)	=SQRT(G43*(1-G43)/$E$44)	=F56/G56

Panel 2. Excel formulas.

Figure 3. Illustration of second-digit test. Illustration of numerical results (panel 1) and Excel formulas[15] (panel 2) for a mock dataset.

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[15] The formula for Z-score (Nigrini, 2012) is (Absolute value(Observed Percentage – Expected Percentage) – (1/2N))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) > (1/2N), where N is the number of data items or (Absolute value(Observed Percentage – Expected Percentage))/(SQRT((Expected Percentage (1-Expected Percentage)/N)), if (Observed Percentage – Expected Percentage) < (1/2N),), where N is the number of data items.